MAT138H1 HW1, DUE 9/26/2022

(1) (JDH 1.3) Prove that $2^{1/4}$ is irrational, both directly and as a consequence of the fact that $\sqrt{2}$ is irrational.
(2) Is the product of any two irrational numbers irrational? Prove it or give a counterexample.
(3) A prime number is a positive integer $p > 1$ such that p is divisible only by 1 and itself. Prove that the square root of a prime number is irrational.
(4) (JDH 1.10) Criticize this proof:

False Claim 1. *$\sqrt{14}$ is irrational.*

Proof. $\sqrt{14} = \sqrt{7} \cdot \sqrt{2}$, and we know by (3) that $\sqrt{7}$, $\sqrt{2}$ are irrational, as 2 and 7 are both prime. Hence $\sqrt{14}$, being the product of two irrational numbers, is irrational. □

(5) Let x be an infinitely repeating decimal, i.e. $x = 0.a_1a_2\cdots a_da_1a_2\cdots a_d\overline{a_1a_2\cdots a_d}$. Prove that x is a rational number.
(6) (JDH 2.7) Prove that the product of any four consecutive integers is a multiple of 24.
(7) (JDH 2.5) Prove that the product of any k consecutive integers is always a mutliple of k.
(8) Prove that $n^3 - n$ is a multiple of 6 for any integer n.